FLOW CHARACTERISTICS OF DILUTE SUSPENSIONS WITH RIGID ELLIPSOIDAL PARTICLES IN AN EXTERNAL FORCE FIELD

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The effect of an electric field on the rheological behavior of dilute suspensions with rigid ellipsoidal dielectric particles is analyzed.

On the basis of the structural-continuum approach, the authors in [1, 2] have derived the rheological equations of state for dilute suspensions with rigid ellipsoidal (ellipsoid of revolution) particles, considering not only their rotational Brownian movement but also the effect of external force fields on their kinematic orientation. A basic difficulty in using these equations for practical calculations is the necessity of finding the distribution function of orientation angles of the rotation axes. In several other studies [3-7] this distribution function was determined with the aid of asymptotic expansions in powers of the presumably small parameters K/D_r and $(\chi_1-\chi_2)E^2/D_r f_r$, these results being of little interest, however, inasmuch as the said parameters may not be small where the rheological behavior under real flow conditions of a suspension with nearly Newtonian characteristics is concerned.

In this study we will use Peterlin's method [8], which yields the distribution function also for cases other than those considered in [3-7]. We will use already known calculations for the effective viscosity and the normal stress differences as functions of the shearing rate and of the field intensity.

Let us consider a simple shear flow:

$$v_x = v_z = 0, \qquad v_y = Kx, \qquad K = \text{const}$$

of a suspension with rigid dielectric ellipsoids in an electric field with the intensity

$$E_x = E\cos\alpha, \qquad E_y = E\sin\alpha, \qquad E_z = 0, \qquad E = \text{const}.$$

The distribution function F, in spherical coordinates, satisfies the following equation [8]:

$$\frac{\partial F}{\partial t} = D_r \Delta F - \operatorname{div}(\vec{\omega}F). \tag{1}$$

The equation or orientation [1, 2] yields for our case the following components of angular velocity

$$\omega_\varphi = \dot{\varphi} = \frac{K}{2}(1 + R\cos 2\varphi) - \frac{\varkappa D_r}{2}\sin 2\varphi',$$
$$\omega_\theta = \dot{\theta} = \frac{KR}{4}\sin 2\varphi \sin 2\theta + \frac{\varkappa D_r}{4}(1 + \cos 2\varphi')\sin 2\theta. \tag{2}$$

As in the case E = 0 [8], we will also seek here the solution to Eq. (1) in the form of an asymptotic expansion in powers of the parameter R whose absolute value is not larger than unity:

$$F(\varphi', \theta) = \sum_{j=0}^{\infty} R^j \left[\frac{1}{2}\sum_{n=0}^{j} a_{n0,j} P_{2n}(\cos\theta) + \sum_{n=1}^{j}\sum_{m=1}^{n} (a_{nm,j}\cos 2m\varphi' + b_{nm,j}\sin 2m\varphi') P_{2n}^{2m}(\cos\theta)\right]. \tag{3}$$

Inserting (2) and (3) into (1), we obtain the following recurrence relations for the coefficients of expansion (3) when $\partial F/\partial t = 0$:

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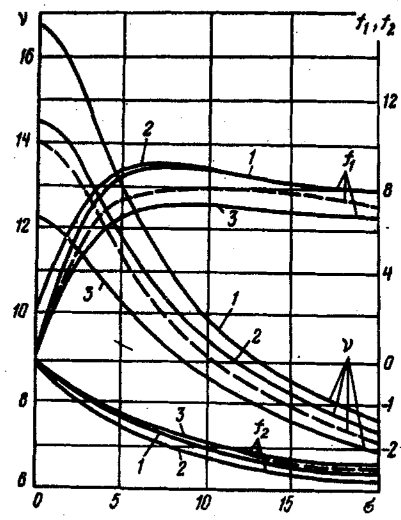

Fig. 1. Curves of ν, f_1, and f_2 vs σ for: 1) α = 0; 2) α = π/4; 3) α = π/2.

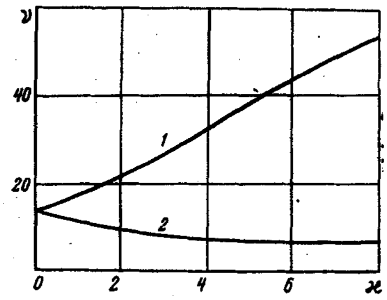

Fig. 2. Curves of ν vs ϰ.

$$-n(2n+1)a_{n0,j} = \sigma\cos 2\alpha A(n,\ 0,\ j-1;\ b) + (\beta+\sigma\sin 2\alpha)A(n,\ 0,\ j-1;\ a) + \frac{\beta}{4}C(n,\ 0,\ j-1;\ a),$$

$$b_{n0,j} = 0,$$

$$-2n(2n+1)a_{nm,j} - \sigma m b_{nm,j} = \sigma\cos 2\alpha\,[A(n,\ m,\ j-1;\ b) + B(n,\ m,\ j-1;\ b)] + (\beta+\sigma\sin 2\alpha)\,[A(n,\ m,\ j-1;\ a) - B(n,\ m,\ j-1;\ a)] + \frac{\beta}{2}C(n,\ m,\ j-1;\ a),$$

$$\sigma m a_{nm,j} - 2n(2n+1)b_{nm,j} = -\sigma\cos 2\alpha\,[A(n,\ m,\ j-1;\ u) + B(n,\ m,\ j-1;\ a)] + (\beta+\sigma\sin 2\alpha)\,[A(n,\ m,\ j-1;\ b) - B(n,\ m,\ j-1;\ b)] + \frac{\beta}{2}C(n,\ m,\ j-1;\ b),$$

where

$$a_{00,0} = \frac{1}{2\pi};\quad \beta = \frac{\varkappa}{R};\quad A(n,\ m,\ j;\ x)$$

$$= \frac{1}{4}\Big[-\frac{(2n+1)(2n-2m-3)(2n-2m-2)(2n-2m-1)(2n-2m)}{(4n-3)(4n-1)} \times x_{n-1m+1,j} + \frac{3(2n-2m-1)(2n-2m)(2n+2m+1)(2n+2m+2)}{(4n-1)(4n+3)}x_{nm+1,j} + \frac{2n(2n+2m+1)(2n+2m+2)(2n+2m+3)(2n+2m+4)}{(4n+3)(4n+5)}x_{n+1m+1,j}\Big];$$

$$B(n,\ m,\ j;\ x) = \frac{1}{4}\Big[\frac{2n+1}{(4n-3)(4n-1)}x_{n-1\,m-1,j} - \frac{3}{(4n-1)(4n+3)}x_{nm-1,j} - \frac{2}{(4n+3)(4n+5)}x_{n+1m-1,j}\Big];$$

$$C(n,\ m,\ j;\ x) = \frac{(2n+1)(2n-2m-1)(2n-2m)}{(4n-3)(4n-1)}x_{n-1m,j} + \frac{4n^2+2n-12m^2}{(4n-1)(4n+3)}x_{nm,j} - \frac{2n(2n+2m+1)(2n+2m+2)}{(4n+3)(4n+5)}x_{n+1m,j}.$$

On the basis of the rheological equations of state for the given medium [1, 2] and with the aid of a computer, formulas (4) yield the effective viscosity $\mu = \mu_0(1+\Phi\nu)$ and the normal stress differences $T_{yy} - T_{zz} = \mu_0\Phi K f_1$ and $T_{xx} - T_{zz} = \mu_0\Phi K f_2$. The results of calculations for ν, f_1, and f_2 as functions of σ and α are shown in Fig. 1, with β = 0.8 and $a/b = 10$. The dashed line represents the case β = 0. The curves in Fig. 2 represent ν as a function of $(\chi_1-\chi_2)E^2/D_r f_r$ for $a/b = 10$, σ = 0, and α = 0 (curve 1) or α = π/2 (curve 2).

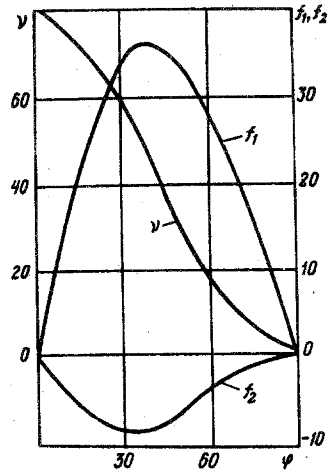

Fig. 3. Curves of ν, f_1, and f_2 vs angle φ (degrees).

In the case of "floating" particles (when the equation of orientation has a stationary solution), the relation found in [10] for ν as a function of the "floating" angle φ (when particles float in the plane of shear) and the rheological equations of state derived for the case in [1, 2] can be used for calculating the relations for f_1 and f_2, as shown in Fig. 3 for $a/b = 10$.

Thus, the superposition of an electric field on a shear flow of a dilute suspension with rigid dielectric ellipsoids will change the kinematic orientation of the suspended particles and this may, depending on the direction of the field, result in an increase or a decrease of the effective viscosity.

The viscosity of the suspension decreases as the electric field parallel to the flow ($\alpha = \pi/2$) increases (the field aids the orientation of suspended particles along the stream lines); when the field is parallel to the viscosity gradient ($\alpha = 0$), then the effective viscosity increases fast.

In all cases the viscosity of a suspension decreases with higher shearing rates.

Dilute suspensions with rigid (and dielectric) ellipsoids in an electric field exhibit non-Newtonian behavior even when the rotational Brownian movement of particles is negligible.

NOTATION

$\sigma = K/D_r$;
$\varkappa = (\chi_1 - \chi_2)E^2/D_r f_r$;
K is the shearing rate;
D_r is the rotational diffusivity;
f_r is the coefficient of rotational friction;
χ_1 and χ_2 are the principal values of the dielectric susceptibility along the axis of rotation of an ellipsoid and along the normal to its direction of the field, respectively;
E_x, E_y, and E_z are the components of the electric field intensity, in Cartesian coordinates OXYZ;
$E^2 = E_x^2 + E_y^2 + E_x^2$;
v_x, v_y, and v_z are the components of velocity;
F is the distribution function;
$\vec{\omega}$ is the angular velocity of an ellipsoid;
$R = (a^2 - b^2)/(a^2 + b^2)$;
$2a$ is the length of the axis of rotation of an ellipsoidal particle;
$2b$ is the equatorial diameter of an ellipsoidal particle;
φ is the angle between the OX axis and the projection of the axis of rotation on the OXY plane;
θ is the angle between the OZ axis and the axis of rotation;
α is the angle between the OX axis and $\vec{E}$;
$\varphi' = \varphi - \alpha$;
P_{2n} are Legendre polynomials of the first kind;
P_{2n}^{2m} are associated Legendre polynomials;
μ_0 is the dynamic viscosity of the solvent;
μ is the viscosity of the suspension;
Φ is the volume concentration of suspended particles;
T_{ij} is the stress tensor;
ν, f_1, and f_2 are functions of σ, β, α, a/b.

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